EXERCISES

Mathematica 6 ∼ Lab Number 1

Problem 1. Evaluate

$$\int_0^\pi \cos\big(x\sin\theta\big)\,d\theta$$

and use **Plot[% , { x, 0, 20 }]** to plot the famous result. Demonstrate that **Plot[Evaluate[$\int_0^\pi \cos\big(x\sin\theta\big)\,d\theta$], { x, 0, 20 }]** does the same job without the distraction of intermediate output.

Convince yourself that *Mathematica* struggles inconclusively if the **Evaluate[ ]** detail is omitted: you will have to abort (**Control-period**) after 30 seconds or so, or the Kernel will exhaust its memory and shut down.

Command *Mathematica* to tell you about **Evaluate**.

Problem 2. Create a link, named "FibonacciBiography," to the Wikipedia website that provides such information.

The Fibonacci numbers are defined recursively

$$F_1 = F_2 = 1 \quad \text{and} \quad F_n = F_{n-1} + F_{n-2} \;:\; n = 3, 4, 5, \ldots$$

and grow very rapidly. We are interested in discovering *how* rapidly. To that end, determine the numerical values of F_{51}/F_{50} and F_{101}/F_{100}.

Looking to the numerical values of F_{101}/F_{100} and F_{100}/F_{101}, we conclude that the asymptotic value of F_{n+1}/F_n is a solution of the equation

$$x = 1 + 1/x$$

Command **Solve[x==1+1/x, x]** to discover the solutions of that equation, evaluate the positive solution and compare that number to the numerical value of **GoldenRatio**. Evidently

$$F_n \sim a \cdot b^n \quad : \quad \text{What is the value of } b?$$

Use $\log F_n - n \log b \sim \log a$ with $n = 50, 100$ to obtain an estimate of the value of a. Look in this connection to the value of $\log(1/\sqrt{5})$.

Use what you now know about the values of a and b to construct 40-place evaluations of the ratios

$$\frac{F_{50}}{a\,b^{50}} \quad \text{and} \quad \frac{F_{100}}{a\,b^{100}}$$

Next construct the generating function

$$\sum_{n=1}^{\infty} \frac{1}{n!} F_n x^n$$

and process the output with the successive commands
Series[%,{x,0,5}]
Simplify[%]

Finally, use

logfib=Table[Log[Fibonacci[k]],{k,1,100}]//N

to construct a list of the logs of the first 100 Fibonacci numbers, and then use **ListPlot** to display that data. Call that figure **FibonacciPoints**. Plot $\log(ab^n) : 0 \leqslant n \leqslant 100$ and call that figure **FibonacciLine**. Superimpose those two figures.

Problem 3. A curve is described parametrically by the equations

$$\begin{aligned} x(t) &= \sin(5t)\cos(2t) \\ y(t) &= \sin(3t)\sin(2t) \end{aligned}$$

Assuming $0 \le t \le 2\pi$, use **ParametricPlot** to display that curve. Do the same after installation of these options:

Axes->None, Frame->True, AspectRatio->Automatic

Do the same after you have changed **Frame->True** to **Frame->False** and installed the option **Ticks->False**.

Problem 4. Let g_1, g_2, g_7 and g_8 be the names assigned to plots (assume $0 \le x \le 2\pi$ and install the option **PlotRange->All**) of the functions

$$\begin{aligned} f_1(x) &= \sin^1 x \\ f_2(x) &= \sin^2 x \\ f_7(x) &= \sin^7 x \\ f_8(x) &= \sin^8 x \end{aligned}$$

Construct a 2×2 composite figure in which g_1, g_2, g_7 and g_8 occupy the NW, NE, SW and SE positions, respectively. How to do so? Ask **?Grid**.

Problem 5. Define

$$f(x) = \sum_{k=1}^{30} \frac{1}{k + k^{\frac{1}{3}}} \sin 2\pi k x$$

and—using **//Timing** to record how long it takes *Mathematica* to do the work—plot that function on the interval $0 < x < 2$.

REMARK: It took *Mathematica* 5.2 a much longer time to produce an inferior result: one had to install options **MaxBend->1** and **PlotPoints->120** to achieve the resolution that *Mathematica* 6 has here achieved automatically.

Problem 6. Construct a contour plot of the function

$$\begin{aligned} f(x,y) = \quad & \frac{1}{(x-1)^2+(y-1)^2} - \frac{3}{(x-1)^2+(y+1)^2} \\ & + \frac{1}{(x+1)^2+(y+1)^2} - \frac{3}{(x+1)^2+(y-1)^2} \end{aligned}$$

Stipulate that $-10 \leqslant x \leqslant 10$ and $-10 \leqslant y \leqslant 10$.

Do the same after installing the option **`PlotPoints->100`**.

Remove that option and install the option **`ContourShading->False`**.

Construct a figure showing the curve that is defined implicitly by the equation

$$x^2 + \tfrac{1}{4}y^2 = 1$$

Stipulate that $-3 \leqslant x \leqslant 3$ and $-3 \leqslant y \leqslant 3$. Turn the frame off, install axes in its place.